Multiple Actuator Support of the 48 Inch Primary Mirror: Force Optimization and Sensitivity to Actuator Force Errors

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August 28, 1996

1 Introduction

This document summarizes the results of a project to design an improved support for the 48 inch telescope's primary mirror. The 48 inch primary is a honeycomb, borosilicate mirror cast at the Steward Observatory Mirror Lab and polished by Rayleigh Optical in Tucson, Arizona. The mirror weighs 335 lbs. The current air support performs inadequately, and investigations by myself, Nelson Caldwell and Daniel Fabricant have shown that it cannot be readjusted for acceptable performance. As a result, the decision to replace the primary mirror support (and cell) has been made. The design of the improved support has been carried out by Bob Fata, myself and Daniel Fabricant.

The axial forces and their positions in the final support concept are summarized in Table 4. This scheme uses 27 axial force actuators and 3 axial hard points. The positions of the actuators, axial hardpoints, and radial hard points are shown in Figures 1 and 2. The forces in Table 4 and Figure 1 are appropriate for the zenith pointing mirror, and must be scaled as the cosine of the zenith angle. Note that the axial hard points carry a significant load, but that the forces on the radial supports should be adjusted so that the radial hard points carry the minimum load.

When the telescope is not zenith-pointing, the radial forces are carried by six push-pull actuators, and the mirror is constrained by three radial hardpoints (see Section 3). The hard-points are assumed to be located at the backsheet of the mirror. The radial forces are assumed to be applied at an axial position in line with the center-of-gravity (CG) of the mirror. The load is then transferred to the top and bottom facesheets of the mirror with a thermally compensated load spreader. The nominal position of the CG of the bare mirror is 2.34 inches below the vertex of the mirror (4.22 inches above the back face), as shown in Figure 3. Once the load spreaders are designed, the CG should be recomputed taking the load spreaders' mass into account. The final axial-force results assume that the radial load spreaders each weigh 1.67 lbs. If the load spreaders have a different weight, we will have to reoptimize the axial forces.

To prevent the mirror from lifting from the axial hard points due to slight axial force errors, we recommend that the minimum weight on the hard points be maintained at 9 lbs each. To be precise, the force should scale as the cosine of the zenith angle until a threshold of 9 lbs is reached. At that point, the weight on the hardpoints should be maintained at 9 lbs. This occurs at a zenith angle of $\sim 60°$.

The axial and radial support forces could be supplied by several types of actuators. We believe that a system using counterweights for the radial forces would be simplest. The axial support could use either counterweights or air actuators controlled by load cells at the axial hard points. In either case, the actuators should be compact so that the honeycomb ventilation holes in the back sheet of the mirror are not blocked. Careful attention must be paid to controlling parasitic forces as summarized in Section 4.3.

Regardless of the type of axial support actuator, accurate load cells at the axial and radial hard points should be provided, as well as a convenient display (in pounds or Newtons) with RS-232 or network interface. Good, inexpensive commercial electronics are available for this purpose.

2 Axial Force Optimization

I have computed a set of optimal actuator forces for supporting the 48 inch mirror using a linear least squares fit that minimizes the slope errors at the mirror surface. The influence functions for each actuator have been calculated by Bob Fata using finite element techniques. This problem is simplified by the fact that the ideal telescope produces a perfect image. Therefore, any slope error at the mirror surface can be easily interpreted as an image blur.

Bob Fata previously arrived at a 36 actuator support that he optimized by minimizing the deflections of the mirror at the actuator positions. Using the linear least squares method, I have been able to reduce the number of actuators to 27 while improving the RMS image diameter from $0\overset{''}{.}16$ to $0\overset{''}{.}13$ RMS diameter.

For my optimization, Bob constructed a finite element model taking advantage of the 60° symmetry of the mirror. He provided me with: (1) the surface slope errors with the mirror supported only on the 3 hard points and subject to an axial gravity load, and (2) the slope errors with the mirror constrained axially at the 3 hard points and subject only to a 1 lb axial force at candidate actuator positions (at mirror rib intersections). The second group of slope errors are the actuator influence functions. More precisely, because of the symmetry of the model, each corresponds to the influence function of 3 or 6 actuators pressing simultaneously with the same force. We began the optimization with a set of 6 actuator locations (in the 60° segment). Initially, we ignore any additional weight associated with the radial counterweights.

Our goal is to determine the forces at the hardpoints and actuators required to produce the minimum final slope errors. This is a linear least squares problem, where the linear coefficients are the forces applied to each actuator and the merit function is the sum of the squares of the residual slope errors. This merit function minimizes the RMS image diameter. I used the General Least Squares method with Singular Value Decomposition described in "Numerical Recipes". The basis set of actuator response slope errors form the 6-column design matrix of the fitting problem. The forces discussed below are for the zenith pointing mirror; we assume that the forces are varied as the cosine of the zenith angle as the telescope is tilted away from the zenith.

The initial set of derived forces was as follows:

Table 1. Optimized 36 Point Support

RMS Image Diameter = $0''.1220$

Actuator	Force (lb)
2646	14.91
2661	0.39
2853	8.31
2982	1.24
3083	8.83
3162	13.56
3317	8.84
3331	7.41
Hard Points	16.24

The next step was to remove actuators 2661 and 2982 and refit since they had rather small forces. This proved to be a successful move causing no degradation in image quality:

Table 2. Optimized 27 Point Support
RMS Image Diameter = $0\farcs1224$

Actuator	Force (lb)
2646	15.15
2853	8.44
3083	8.95
3162	13.84
3317	8.70
3331	7.32
Hard Points	18.04

The next step was to combine the actuators into two groups so that we will have to use only 2 distinct values of force. This was accomplished by adding the slope errors for the members of the group and producing a new data file that has the response of applying a 1 lb force to each of the actuators in the group. The fit was then redone with the new 2-column design matrix.

The result is:

Table 3. 27 Point Support, Two Actuator Support Values
RMS Image Diameter = $0\farcs132$

Actuator	Force (lb)
2646	16.05
2853	7.49
3083	7.49
3162	16.05
3317	7.49
3331	7.49
Hard Points	18.58

Bob entered these axial support forces into the full (360°) finite element model and verified that the surface displacements improved slightly from his original 36 point support. The

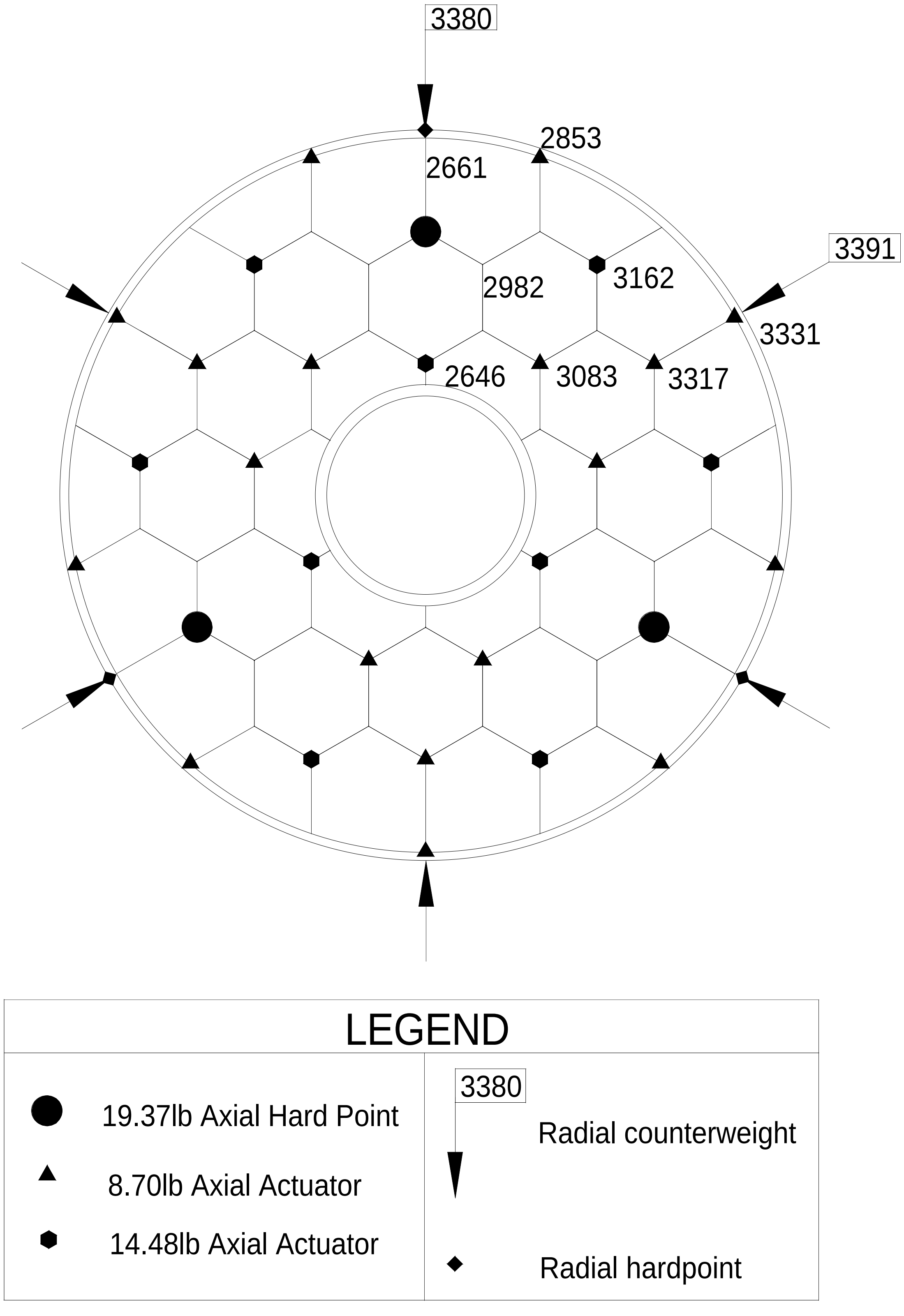

Fig. 1.— Diagram of 48 inch mirror showing final setup of axial and radial forces.

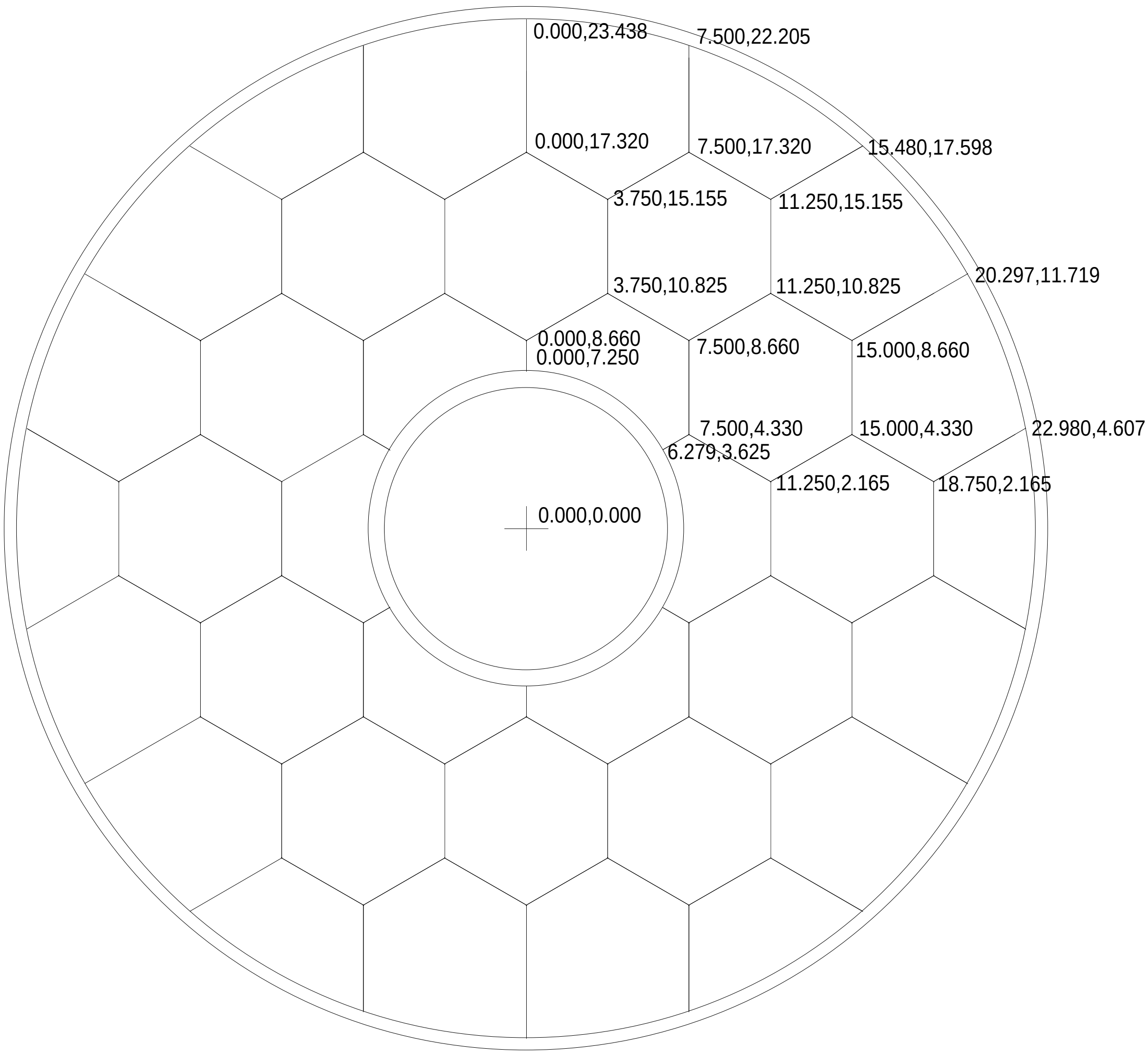

Fig. 2.— Locations of the rib intersections in the 48 inch mirror. Units are inches.

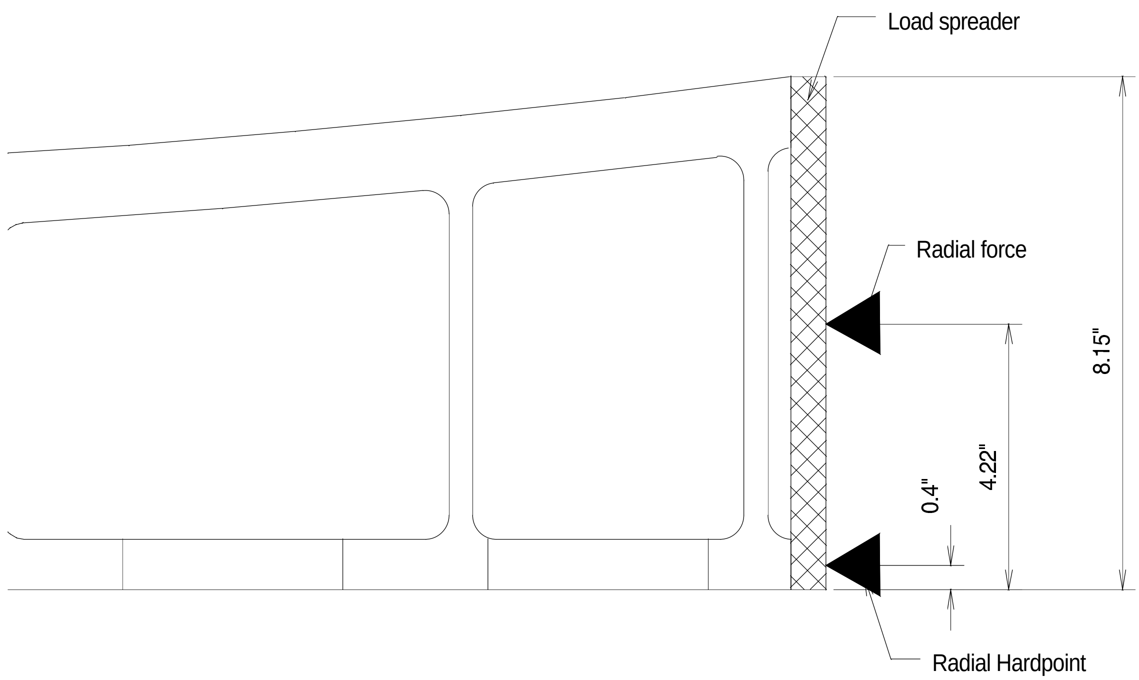

Fig. 3.— Cutaway view of mirror showing locations of radial forces and hardpoints. The representation of the load spreader is schematic. The actual hardware will be thermally matched to the mirror.

spot diagram produced from the finite element slope errors is shown in Figure 4 (top). The locations of the rays in the pupil are shown in Figure 5. We ignore the fact that the rays are not perfectly uniformly distributed in the pupil. The RMS image diameter computed from the full model is $0\farcs128$.

At this point, Bob pointed out that we should include the additional weight of the load spreaders associated with the radial counterweights. These will serve to apply the radial forces to the top and bottom plates of the mirrors. A study of the effect of the additional weight yielded the surprising result that adding additional weight to the edge of the mirror at the six radial counterweight locations improves the image quality in a 27-actuator 2-force system! The optimal additional weight is 1.67 lbs per load spreader, though any weight between 0 and 3.3 lbs is acceptable. The final force values should be redetermined once the load spreader is designed if it does not weigh 1.67 lbs. See Figure 1 for a diagram of the actuator locations and forces.

The optimal system is:

Table 4. 27 Point Support, Two Actuator Support Values, 1.67 lbs per Radial Counterweight

RMS Image Diameter = $0\farcs126$

Actuator	Force (lb)
2646	14.48
2853	8.70
3083	8.70
3162	14.48
3317	8.70
3331	8.70
Hard Points	19.37

Bob supplied me with the response functions for the other 9 possible actuator locations in the 60° model so that I could look for even better solutions. I then ran the least squares fitter on all 32768 combinations of actuators. I found that no solutions with RMS image diameter less than $0\farcs15$ existed for any system with fewer than 24 actuators. There were two 24 acuator systems, but the savings in cost over a 27 actuator system seem minimal and the image quality is somewhat worse ($0\farcs153$ after combining the forces into 2 sets). About a dozen other 27 actuator systems produced similar image quality, but few looked amenable to combining the forces into only two groups. As one might expect, the globally

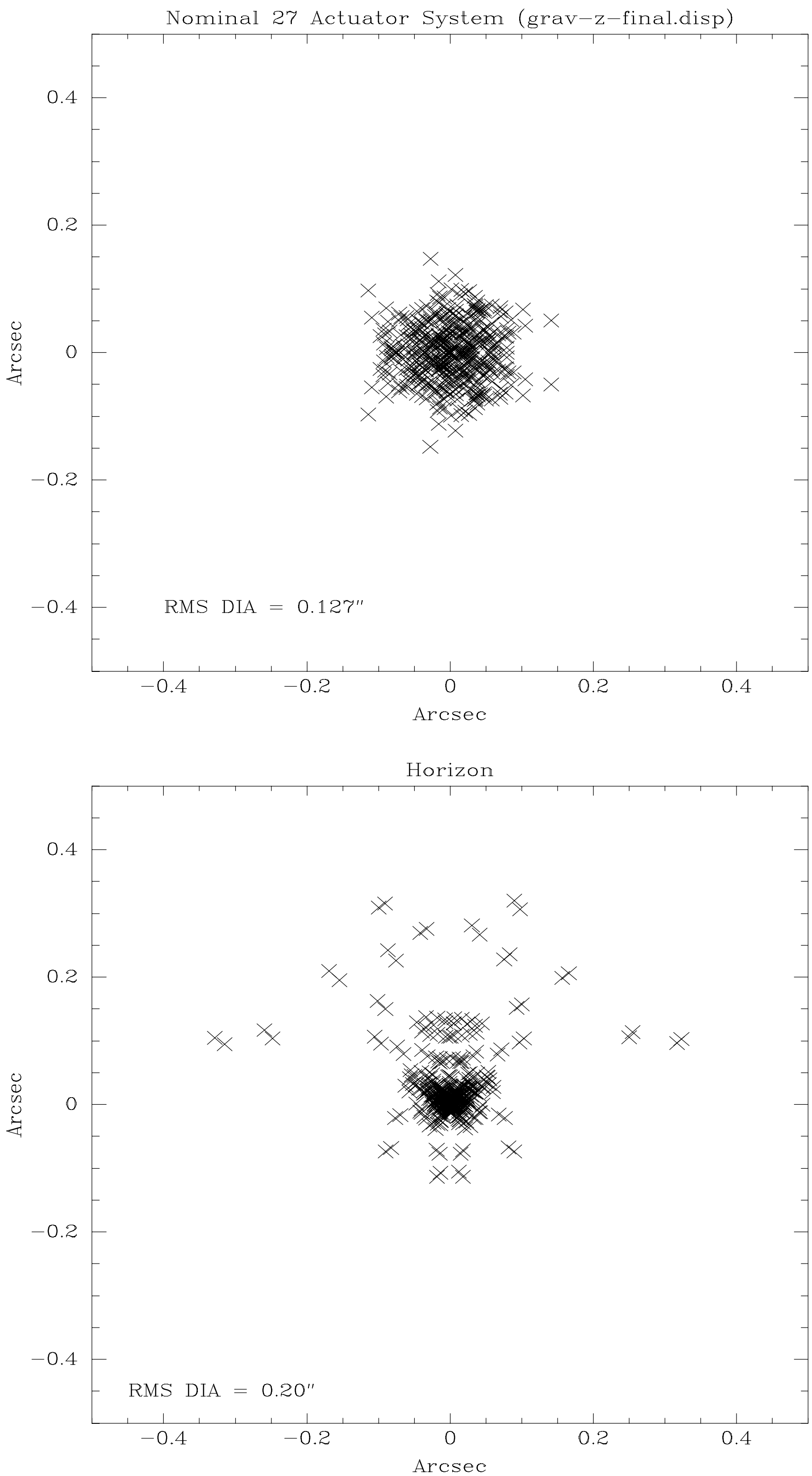

Fig. 4.— Spot diagrams for nominal 27 actuator, zenith pointing (top) and horizon pointing (bottom).

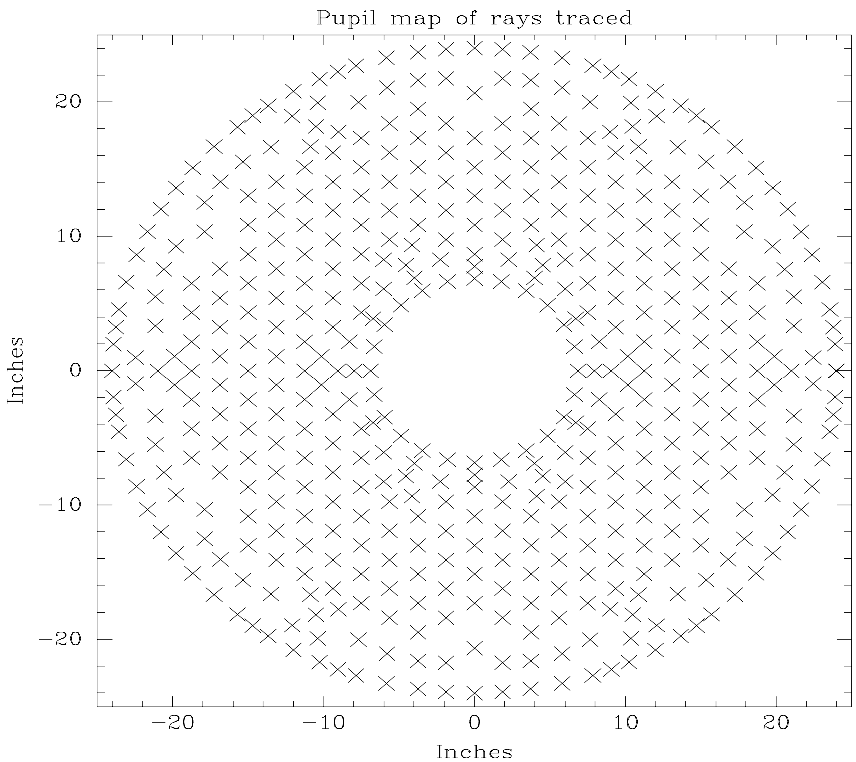

Fig. 5.— Pupil locations of rays traced

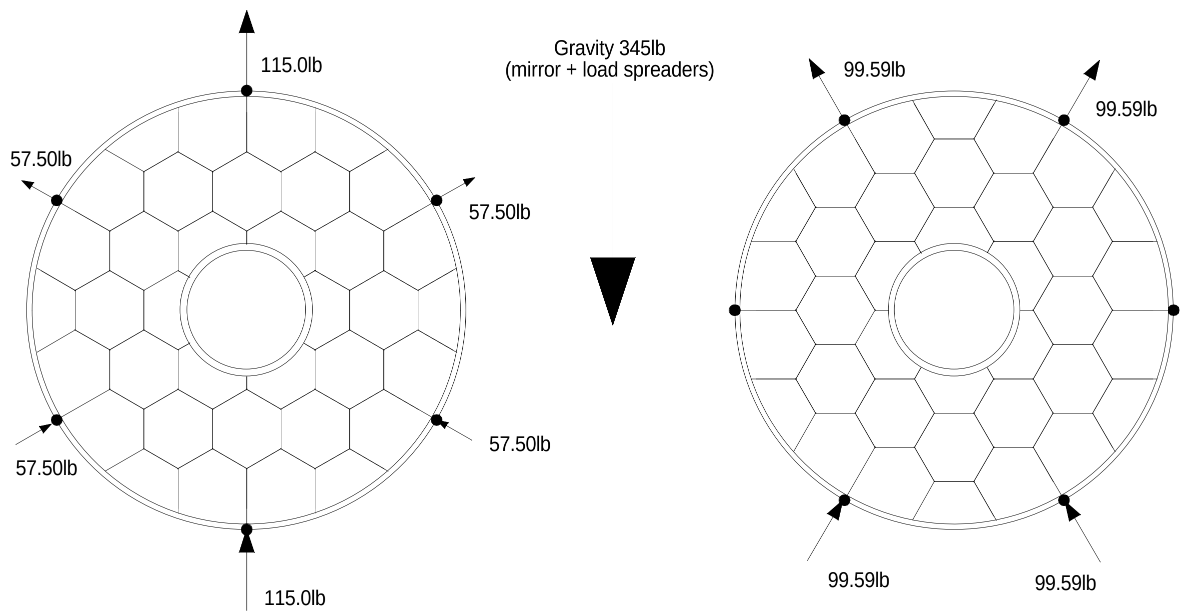

Fig. 6.— Examples of radial forces for two horizon-pointing configurations. In both cases the net force supplied by the actuators cancels gravity.

best solution used all 69 possible actuators but provided only a modest improvement ($0\farcs10$) and at the expense of having some actuators required to produce a downward force. Given the modest returns that could be expected I ended the search.

3 Radial Forces

The radial support of the mirror is applied at six locations at 60° intervals around the mirror, with additional hard points at three of those locations. The forces will be applied with push-pull type counterweights. The maximum force applied at each location will be 1/3 of the combined weight of the mirror and anything bonded to it, or 115.0 lb if the load spreaders weigh 1.67 lbs each. Figure 6 shows two examples of the applied forces when the mirror is horizon pointing.

4 Sensitivities to Force Errors

Bob has run models describing the slope errors produced by 1 lb axial force errors at each of the axial and radial force locations, and for 1 lb radial errors at the radial force locations.

These sensitivity models do not employ the 60° symmetry and so correspond to an error at a single actuator locations. The spot diagrams for these error forces are shown in Figures 7 and 8. These spot diagrams represent the additional ray aberrations introduced by non-ideal forces. The image diameter sensitivities are summarized in Table 5.

Table 5. RMS Image Blur from 1 lb Force Errors.

Node	Actuator type	Force direction	RMS Image Diameter (arcsec)
3162	Axial	Axial	0.029
3317	Axial	Axial	0.032
3331	Axial	Axial	0.049
2646	Axial	Axial	0.015
2853	Axial	Axial	0.029
3083	Axial	Axial	0.020
3380	Radial	Axial	0.015
		Radial	0.005
3391	Radial	Axial	0.049
		Radial	0.005

4.1 Axial actuators

At the zenith, the force errors are caused by errors in the axial actuators. We allow ourselves a total image quality error budget of $0\farcs15$ RMS. Of this $0\farcs13$ is taken up by the perfect mirror. This leaves $0\farcs075$ (subtract in quadrature) for force errors. Allowing equal contributions to add in quadrature from each of the 27 axial actuators, we are allowed $0\farcs015$ per actuator. For the worst-case actuator (3331) this corresponds to a 0.25 lb force error. To make the specifications straightforward, we set the axial-force-error limit to be 0.25 lb at each axial actuator, and also at the hardpoints. These force errors must scale with the cosine of the zenith angle so that they do not contribute when horizon pointing.

We have not computed the sensitivity to radial force errors at the axial actuator locations since the mirror should be extremely stiff in that direction. Since the total force per actuator is less than 20 lb, we can reasonably expect less than 0.5 lb of cross coupled force per actuator which will produce negligible deformation.

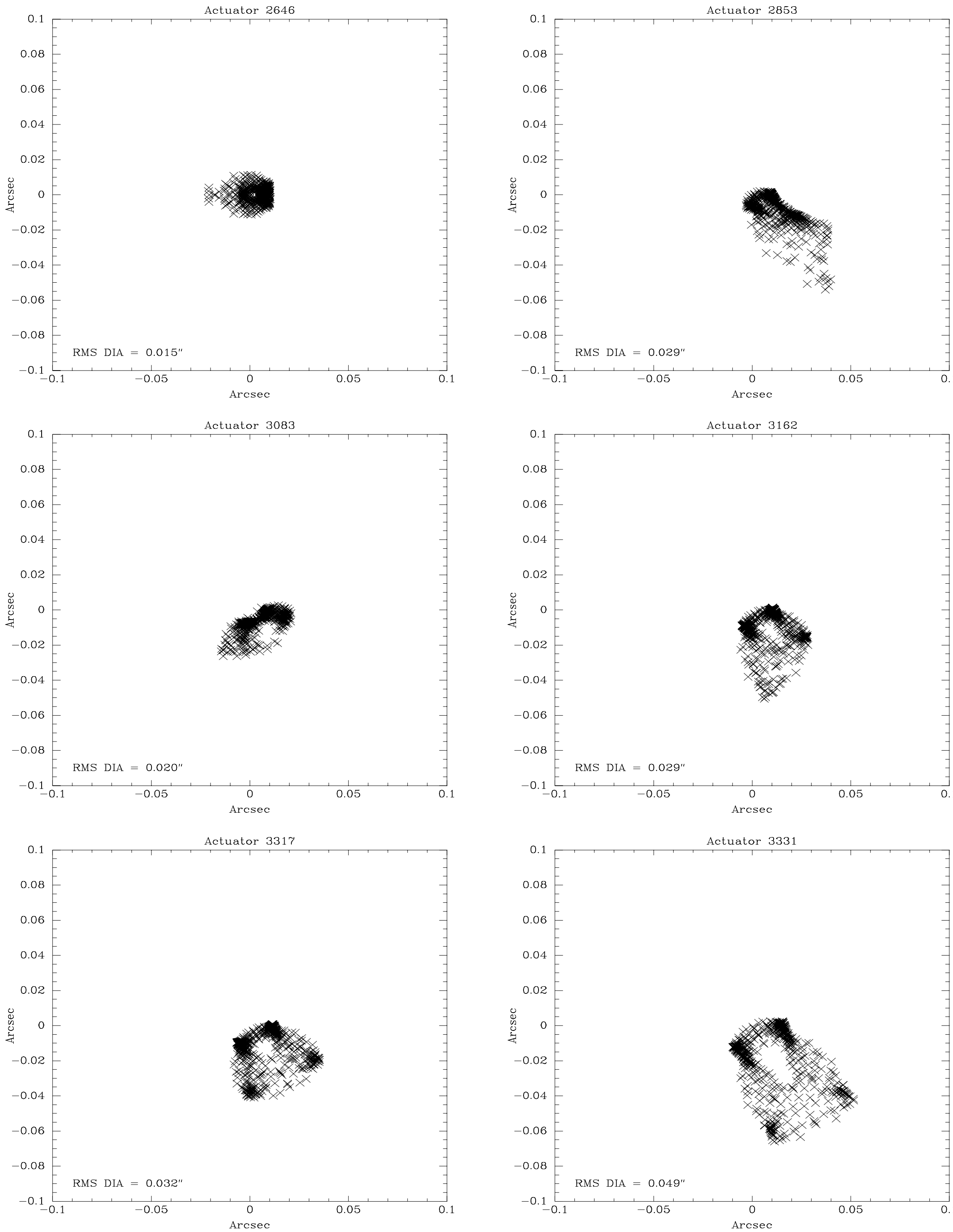

Fig. 7.— Spot diagrams for 1lb axial force errors at axial actuator locations.

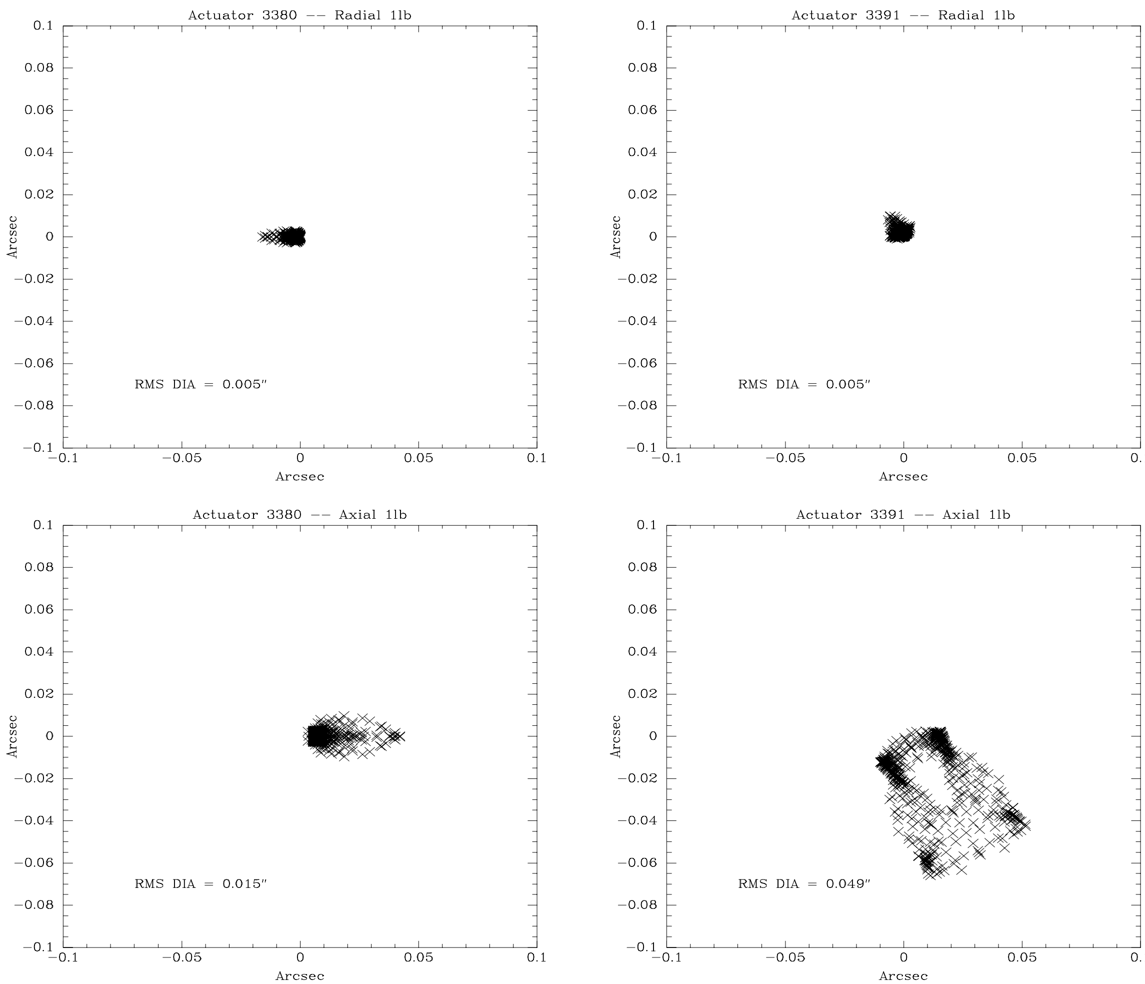

Fig. 8.— Spot diagrams for 1 lb errors at radial actuator locations. Radial force errors are shown on top, axial errors at the bottom.

4.2 Radial actuators

At the radial counterweight locations and hardpoints we can get force errors in both the radial and axial directions. Radial forces result from errors in the amplitude of the force applied. Axial forces result from cross-coupling in the radial counterweights into the axial direction.

When the telescope is horizon pointing, the ideal support system will produce images with RMS diameter of $0''.20$ (see Fig 4). We allow a total of $0''.25$ at the horizon, and divide the error budget evenly between axial and radial force errors. This gives $0''.11$ allowed for each type of error. To be conservative we assume that the error forces will add linearly among the six actuators rather than quadratically. This leads to an allowable error of 3.4 lb radial force per actuator at the horizon. The greater sensitivity to the axial forces ($0''.049$/lb) leads to a limit of 0.34 lb per actuator for cross coupled forces. These force errors must scale approximately with the sine of the zenith angle so that they do not contribute significantly at the zenith.

4.3 Summary of Allowable Error Forces

		Maximum Error Per Actuator (lbs)	
Actuator Type	Force Direction	Zenith	Horizon
Axial	Axial	0.25	0.1
Axial	Radial	0.25	0.1
Radial	Axial	0.1	0.34
Radial	Radial	0.5	3.4

5 Appendix – Support of the mirror during figuring

Since there seems to be no drawing of how the mirror was supported during figuring, I've decided to include one here for posterity. This information is based on conversations with Nelson Caldwell and Dave Anderson. The mirror was supported on a set of eighteen 8-inch diameter belloframs during polishing. These were set up in two rings of 6 and 12. Figure 9 shows the arrangement as I understand it. During figuring the mirror was rotated on top of the support system to various positions. Final hand figuring was done with the mirror in its cell. According to Dave, up to half a wave may have been removed with it in the cell, some of which may have been astigmatism.

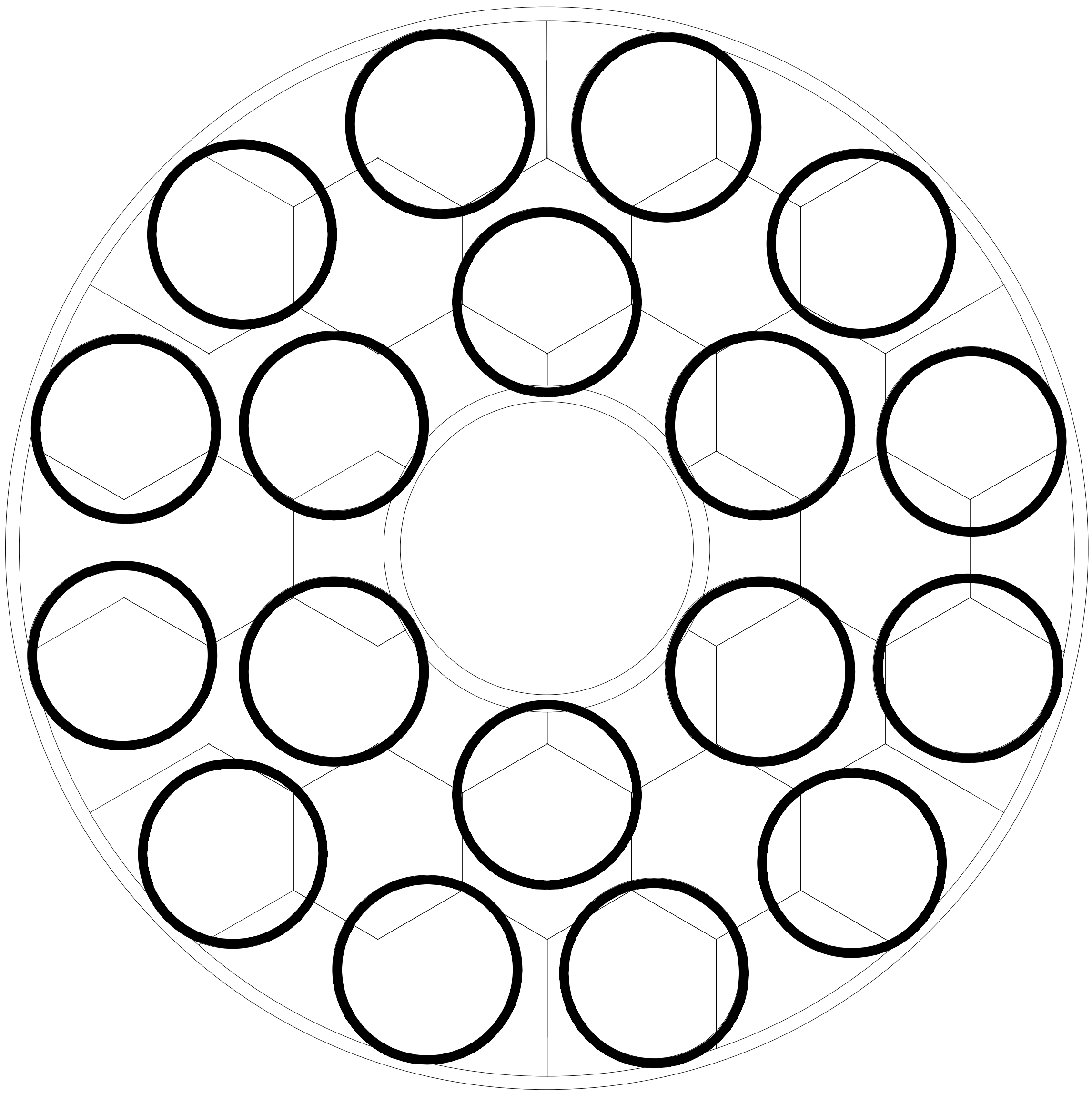

Fig. 9.— Bellofram support configuration used during figuring of the 48” mirror